CONIC SECTIONS COMPASS (CSC)

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ABSTRACT: Although the theoretical principles underlying the construction of conic sections are widely known, simple and fairly inexpensive devices for drawing such sections for continuous increment range are not available in the market. In the present paper a compass specially designed and manufactured to these purposes is explained.

1. INTRODUCTION

The existing templates for drawing circles and ellipses are designed to draw only a selected intermittent set of sizes. Other conic sections, such as hyperbolas and parabolas, may be drawn using the French irregular curve templates. These drawing procedures are therefore, less accurate, inefficient and slow.

A set of drawing instrument (CSC), with which conic sections of continuously varying dimensions could be drawn with adequate accuracy and efficiency is specially designed and manufactured. Depending on the size range the whole set may be contained in box sizes similar to those of the ordinary drawing instruments.The compass is composed of three parts: an Axis,a Support for this axis and a Pen. It is expected that with these simple designs and few components made by a less sophisticated production method, the total cost would be affordable .

2. THEORY AND DRAWING METHODS

2.1 ELLIPSES

By definition, an ellipse, a closed planar curve, is the loci of a set of points on a plane, wherein their sum of distances from any two fixed points on that plane is a constant of $2a=AA'$ (Fig .1). The two specified points, the focals, are spaced a focal distance of $FF'=2c$ apart.

The eccentricities, $e=\frac{c}{a} \leq 1$. However, $e=0$ and $e=1$ represent a circle and a line, respectively.

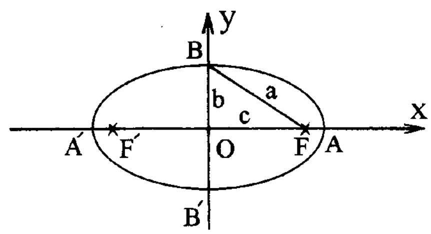

Fig .1. Features of an Ellipse

Accordingly, we have $a^2=b^2+c^2$ and the equation for an ellipse in a rectangular coordinate system can be written as $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

An ellipse may be obtained by intersecting an oblique plane with a right circular cone or cylinder. However, for simplicity, the right circular cylinder has been chosen for drawing an ellipse.In order to draw an ellipse, half of the smaller diameter 2b, is chosen on the pen.An appropriate lenght L is chosen on the Support in order to obtain the greater diametr 2a.The diameter 2b is equal to the diameter of the cylinder. Fig.2. shows the method of drawing an ellipse.

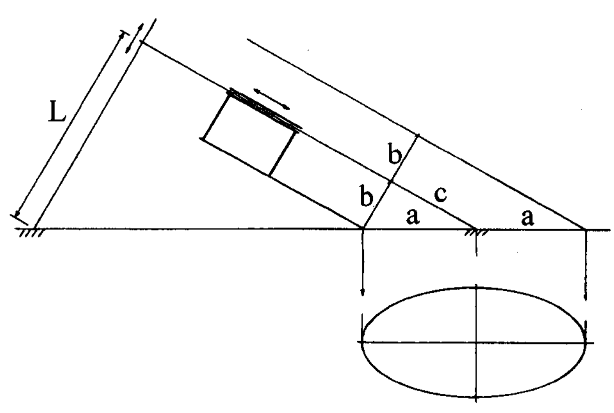

Fig .2. Constraction of Ellipses of Variable a & b.

Since,the variations of the Support lenght L versus $\frac{a}{b}$ gives a continuous curve,the Support might be graduated which facilitates the drawing of continuously varying ellipses of different a and b. With this compass ellipses can be drawn whose diameter ratio are in the range of 1 to 5 , which is much larger than commercially available,intermittent set of templates in the market. Fig.3. shows few samples of ellipses drawn by the CSC .

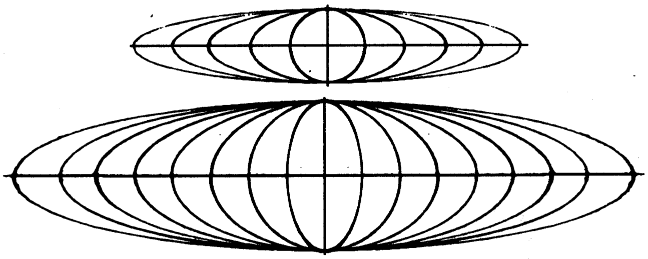

Fig.3. Some Samples of the Ellipses Drawn by the CSC.

2.2 HYPERBOLAS

By definition, a hyperbola is the loci of all the points on a plane, wherein the difference of their distances from any two fixed points (the focals F & F´ ,so that FF´=2c) of that plane is a constant equal to the transverse axis of the hyperbola (AA´=2a).

The eccentricities, $e=\frac{c}{a}$, is always greater than one . But, when it is approaching one, the curve leads to two coaxial lines, whereas at larger values it nears the bisecting normal to FF´ (Fig.4).

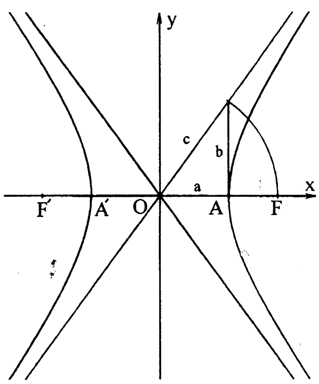

Fig.4. Features of a Hyperbola.

Accordingly, we have $b^2=c^2-a^2$ and the equation becomes $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$. When a plane and a right circular cone intersect each other, depending on their orientations, an ellipse, a hyperbola or a parabola is obtained. A hyperbola is resulted when the plane cuts the generatrice of the cones at both sides of the common apexes. Obviously, the intersection of a specific cone and various planes would produce various hyperbolas.But, for simplicity, consider that the axis of the cone is parallel to the plane of drawing paper. Fig.5. shows the mechanism devised for drawing hyperbolas,$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ generated by a plane parallel to the axis of an appropriate right circular cone at a distance b from that axis.

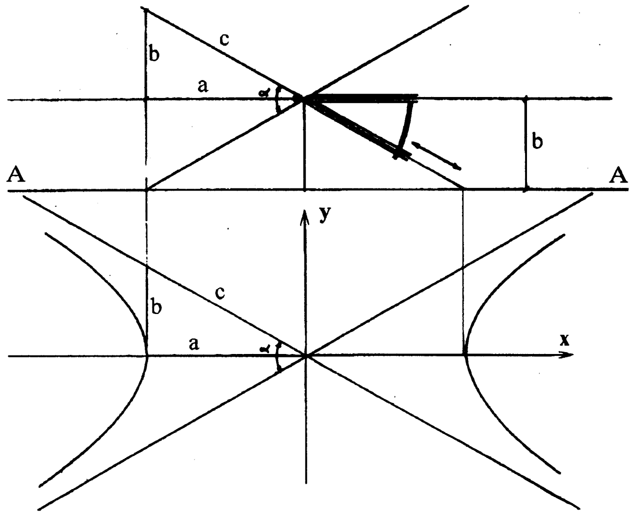

Fig.5. Drawing Method of Hyperbolas.

Here, a cone with the opening α cut by the plane A-A, and the hyperbola drawn are shown in vertical and horizontal views, respectivly. For drawing a hyperbola ,according to the above ,an opening α is chosen on the graduated pen, then a distance b is set on the graduated Support.The sliding pen can draw successively the two curves. Fig.6. shows some samples of hyperbolas drawn by the CSC.

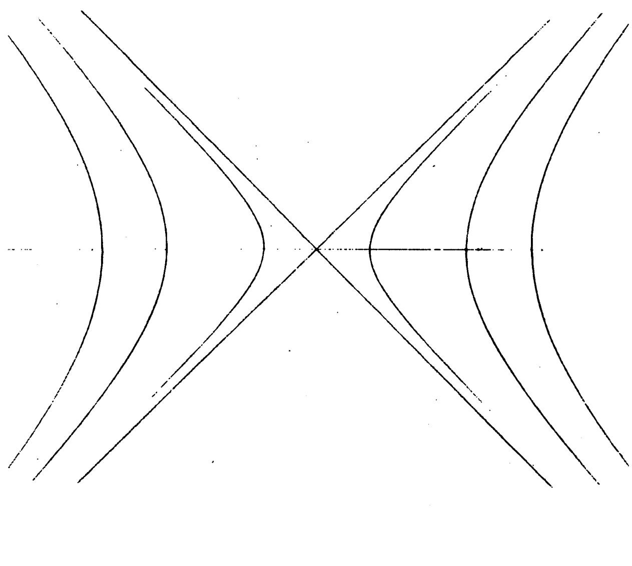

Fig.6. Some Samples of Hyperbolas Drawn by the CSC.

2.3 PARABOLAS

By definition, the loci of all the points of a plane for which the ratios of their distances from a fixed point(the focus, F) and a fixed line, D (the directrix) from that plane are equal to one, is called a parabola. The normal distance of the fixed point and the line is called the parameter p of parabola (Fig.7). Accordingly, the equation becomes $y^2=2px$.

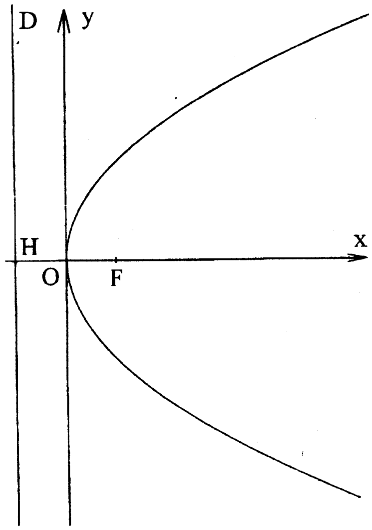

Fig.7. Features of a Parabola.

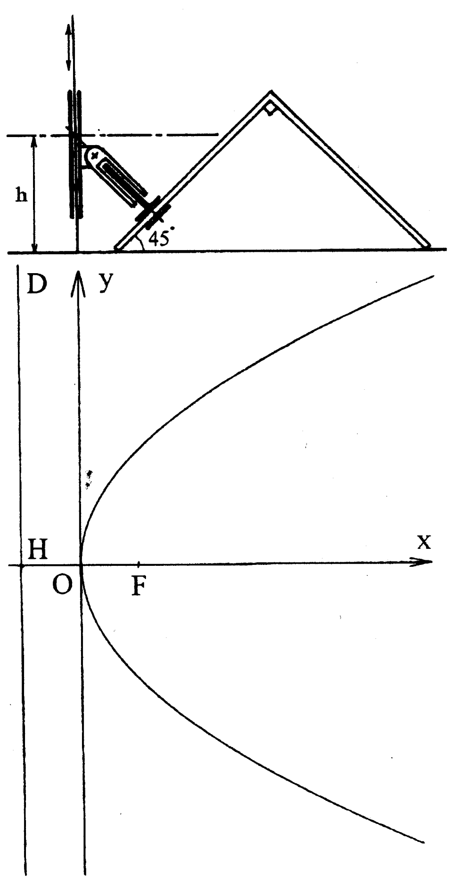

Fig.8. Drawing Method of Parabolas.

From the above definition, it is clear that all parabolas are similar. Hence, the intersection of any specified conic surface with various planes would generate a whole set of parabolas.

In drawing a parabola it can be proved that $p=h.\tan\frac{\alpha}{2}$ where h is the height of the cone apex and α is the apex angle of the right circular cone. When $\alpha=90°$, p=h . Fig.8. shows the method of drawing a parabola. Fig.9. shows some samples of parabolas drawn by the CSC.

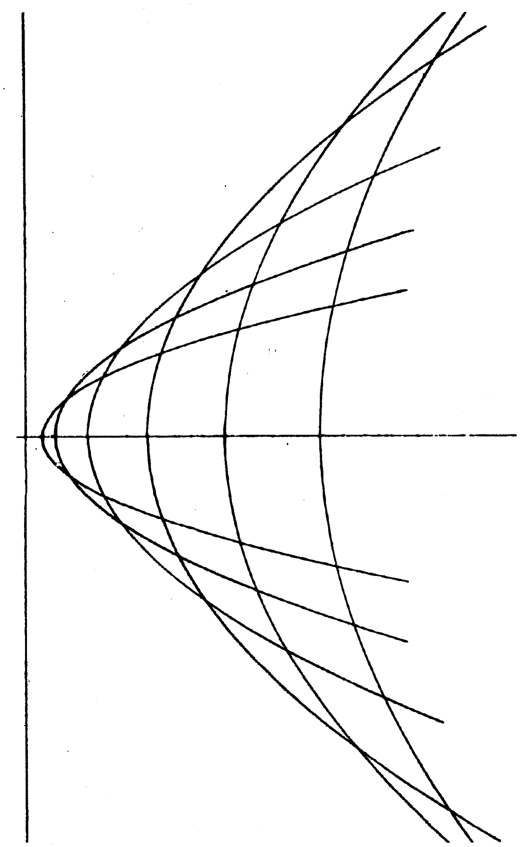

Fig 9. Some Samples of Parabolas Drawn by the CSC.

3. CONCLUSIONS

In this paper a drawing instrument specially designed to draw various conic sections is described. This drawing instrument, though simple but is an advanced tool to draw any specified continuous size range of conic sections. A set of prototype compass has been designed and manufactured by the present author.Therefore we may conclude as follows:

1) this is a very handy and simple tool for drawing any conic section size.
2) it enables to draw any size continuously.
3) complex sections like hyperbolas and parabolas are drawn as easy as ellipses.